

# Some Bianchi Type III String Cosmological Models with Bulk Viscosity

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Published online: 11 May 2007  
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**Abstract** We investigate the integrability of cosmic strings in Bianchi III space-time in presence of a bulk viscous fluid by applying a new technique. The behavior of the model is reduced to the solution of a single second order nonlinear differential equation. We show that this equation admits an infinite family of solutions. Some physical consequences from these results are also discussed.

**Keywords** Cosmic string · Viscous models · Integrable

## 1 Introduction

In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe [10]. Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [9–11, 25, 34, 35]. It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies [36]. These cosmic strings have stress-energy and couple to the gravitational field, it may be interesting to study the gravitational effects that arise from strings.

The general relativistic treatment of strings was initiated by Letelier [14, 15] and Stachel [22]. Letelier [14] has obtained the solution to Einstein's field equations for a cloud

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of strings with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi I and Kantowski–Sachs space-times. Benerjee et al. [5] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty [8], Tikekar and Patel [23, 24]. Patel and Maharaj [18] investigated stationary rotating world model with magnetic field. Ram and Singh [19] obtained some new exact solutions of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [7]. Exact solutions of string cosmology for Bianchi type II, VI<sub>0</sub>, VIII and IX space-times have been studied by Krori et al. [12] and Wang [26]. Singh and Singh [21] investigated string cosmological models with magnetic field in the context of space-time with  $G_3$  symmetry. Singh [20] has also studied string cosmological models with electromagnetic field in Bianchi type II, VIII and IX space-times. Lidsey, Wands and Copeland [16] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Baysal et al. [6] have investigated the behavior of a string in the cylindrically symmetric inhomogeneous universe. Bali et al. [1, 3, 4] have obtained Bianchi types I and IX string cosmological models in general relativity. Yavuz [32] have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Recently Kaluza–Klein cosmological solutions are obtained by Yilmaz [33] for quark matter coupled to the string cloud in the context of general relativity.

On the other hand, the matter distribution is satisfactorily described by perfect fluids due to the large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that when neutrino decoupling occurs, the matter behaves as a viscous fluid in an early stage of the universe. Viscous fluid cosmological models of early universe have been widely discussed in the literature.

Tikekar and Patel [23], following the techniques used by Letelier and Stachel, obtained some exact Bianchi III cosmological solutions of massive strings in presence of magnetic field. Maharaj et al. [17] have generalized the previous solutions obtained by Tikekar and Patel [23] by considering Lie point symmetries. Recently Yadav et al. [31] have studied some Bianchi type I viscous fluid string cosmological models with magnetic field. Recently Wang [26–30] has discussed LRS Bianchi type I and Bianchi type III cosmological models for a cloud string with bulk viscosity. Motivated the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the new integrability of cosmic strings in Bianchi III space-time in presence of a bulk viscous fluid by applying a new technique.

## 2 The Metric and Field Equations

We consider the space-time of general Bianchi III type with the metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2ax}dy^2 + C^2(t)dz^2, \quad (1)$$

where  $a$  is constant. The energy momentum tensor for a cloud of string dust with a bulk viscous fluid of string is given by Letelier [14] and Landau and Lifshitz [13]

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi v_i^l (g_l^j + v_l v^j), \quad (2)$$

where  $v_i$  and  $x_i$  satisfy condition

$$v^i v_i = -x^i x_i = -1, \quad v^i x_i = 0. \tag{3}$$

In (2)  $\rho$  is the proper energy density for a cloud string with particles attached to them,  $\lambda$  is the string tension density,  $v^i$  is the four-velocity of the particles and  $x^i$  is a unit space-like vector representing the direction of string. If the particle density of the configuration is denoted by  $\rho_p$ , then we have

$$\rho = \rho_p + \lambda. \tag{4}$$

The Einstein’s field equations (in gravitational units  $c = 1, G = 1$ ) read as

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j, \tag{5}$$

where  $R_i^j$  is the Ricci tensor;  $R = g^{ij} R_{ij}$  is the Ricci scalar. In a co-moving co-ordinate system, we have

$$v^i = (0, 0, 0, 1), \quad x^i = (0, 0, 1/C, 0). \tag{6}$$

The field equations (5) with (2) subsequently lead to the following system of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = 8\pi \xi \theta, \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = 8\pi \xi \theta, \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{a^2}{A^2} = 8\pi (\lambda + \xi \theta), \tag{9}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{a^2}{A^2} = 8\pi \rho, \tag{10}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0, \tag{11}$$

where the suffix 4 at the symbols  $A, B$  and  $C$  denotes ordinary differentiation with respect to  $t$ . The particle density  $\rho_p$  is given by

$$8\pi \rho_p = \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} \tag{12}$$

in accordance with (4).

The velocity field  $v^i$  as specified by (6) is irrotational. The scalar expansion  $\theta$  and components of shear  $\sigma_{ij}$  are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{13}$$

$$\sigma_{11} = \frac{A^2}{3} \left[ \frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} \right], \tag{14}$$

$$\sigma_{22} = \frac{B^2 e^{-2ax}}{3} \left[ \frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C} \right], \tag{15}$$

$$\sigma_{33} = \frac{C^2}{3} \left[ \frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B} \right], \tag{16}$$

$$\sigma_{44} = 0. \tag{17}$$

Therefore

$$\sigma^2 = \frac{1}{3} \left[ \frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{CA} \right]. \tag{18}$$

### 3 Solutions of the Field Equations

The field equations (7–11) are a system of five equations with six unknown parameters  $A, B, C, \rho, \lambda$  and  $\xi$ . One additional constraint relating these parameters is required to obtain explicit solutions of the system. We assume that the expansion ( $\theta$ ) in the model is proportional to the eigenvalue  $\sigma_2^2$  of the shear tensor  $\sigma_i^j$ . This condition leads to

$$B = \alpha(AC)^\beta, \tag{19}$$

where  $\alpha$  and  $\beta$  are arbitrary constants. Equations (11) leads to

$$A = mB, \tag{20}$$

where  $m$  is an integrating constant. From (19) and (20), we obtain

$$B = MC^N, \tag{21}$$

where

$$M = \alpha^{\frac{1}{1-\beta}} m^{\frac{\beta}{1-\beta}}, \quad N = \frac{\beta}{1-\beta}. \tag{22}$$

By the use of (20) in field equations (8–10) reduce to

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{a^2}{m^2 B^2} = 8\pi(\lambda + \xi\theta), \tag{23}$$

$$\frac{B_4^2}{B^2} + \frac{B_4 C_4}{BC} - \frac{a^2}{m^2 B^2} = 8\pi\rho. \tag{24}$$

Using (13) and (21) in (7), we have

$$(N + 1) \frac{C_{44}}{C} + N^2 \frac{C_4^2}{C^2} = 8\pi\xi(2N + 1) \frac{C_4}{C}. \tag{25}$$

Let us consider

$$C_4 = f(C). \tag{26}$$

Using (26) in (25), we get

$$\frac{df}{dC} + \left\{ \left( \frac{N^2}{N+1} \right) \frac{1}{C} \right\} f = 8\pi\xi \left( \frac{2N+1}{N+1} \right). \tag{27}$$

After integration, (27), reduces to

$$f = 8\pi\xi \frac{(2N+1)}{(N^2+N+1)} C + \frac{P}{C \left( \frac{N^2}{N+1} \right)}, \tag{28}$$

where  $P$  is an integrating constant. Integrating (28), we obtain

$$C = \frac{1}{\xi^{k_4}} [k_1 + k_2 e^{k_3 \xi t}]^{k_4}, \tag{29}$$

where  $S$  is an integrating constant. Therefore

$$B = \frac{M}{\xi^{k_5}} [k_1 + k_2 e^{k_3 \xi t}]^{k_5}, \tag{30}$$

$$A = \frac{mM}{\xi^{k_5}} [k_1 + k_2 e^{k_3 \xi t}]^{k_5}, \tag{31}$$

where

$$\begin{aligned} k_1 &= -\frac{P(N^2+N+1)}{8\pi(2N+1)}, \\ k_2 &= \frac{S}{8\pi(2N+1)}, \\ k_3 &= \frac{8\pi(2N+1)}{(N+1)}, \\ k_4 &= \frac{(N+1)}{N^2+N+1}, \\ k_5 &= \frac{N(N+1)}{N^2+N+1}. \end{aligned} \tag{32}$$

Hence the metric (1) reduces to the form

$$\begin{aligned} ds^2 &= -dt^2 + m^2 M^2 \left[ \frac{k_1 + k_2 e^{k_3 \xi t}}{\xi} \right]^{2k_5} dx^2 + M^2 e^{-2ax} \left[ \frac{k_1 + k_2 e^{k_3 \xi t}}{\xi} \right]^{2k_5} dy^2 \\ &+ \left[ \frac{k_1 + k_2 e^{k_3 \xi t}}{\xi} \right]^{2k_4} dz^2. \end{aligned} \tag{33}$$

Using the suitable transformation

$$\begin{aligned}\frac{k_1 + k_2 e^{k_3 \xi t}}{\xi} &= L \frac{\sin(\xi \tau)}{\xi}, \\ mL^{k_5} x &= X, \\ ML^{k_5} y &= Y, \\ L^{k_4} z &= Z,\end{aligned}\quad (34)$$

the metric (33) reduces to

$$\begin{aligned}ds^2 &= -\left(\frac{L \cos(\xi \tau)}{k_3 \{k_1 - L \sin(\xi \tau)\}}\right)^2 d\tau^2 + \left(\frac{\sin(\xi \tau)}{\xi}\right)^{2k_5} dX^2 \\ &+ e^{-\frac{2aX}{mML^{k_5}}} \left(\frac{\sin(\xi \tau)}{\xi}\right)^{2k_5} dY^2 + \left(\frac{\sin(\xi \tau)}{\xi}\right)^{2k_4} dZ^2.\end{aligned}\quad (35)$$

The rest energy ( $\rho$ ), the string tension density ( $\lambda$ ), the particle density ( $\rho_p$ ), expansion ( $\theta$ ) and shear ( $\sigma$ ) for the model (35) are given by

$$8\pi\rho = k_3^2 k_5 (2k_4 + k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]^2 - \left( \frac{a}{mM} \right)^2 \left[ \frac{\xi}{L \sin(\xi \tau)} \right]^{2k_5}, \quad (36)$$

$$\begin{aligned}8\pi\lambda &= -8\pi k_3 \xi (k_4 + 2k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right] + 3k_3^2 k_5^2 \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]^2 \\ &+ 2k_1 k_3^2 k_5 \left[ \frac{\xi^2}{L \sin(\xi \tau)} - \frac{k_1 \xi^2}{L^2 \sin^2(\xi \tau)} \right] - \left( \frac{a}{mM} \right)^2 \left[ \frac{\xi}{L \sin(\xi \tau)} \right]^{2k_5},\end{aligned}\quad (37)$$

$$\begin{aligned}8\pi\rho_p &= 2k_3^2 k_5 (k_4 - k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]^2 \\ &+ \left[ 8\pi \xi k_3 (k_4 + 2k_5) - 2k_1 k_3^2 k_5 \frac{\xi}{L \sin(\xi \tau)} \right] \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right],\end{aligned}\quad (38)$$

$$\sigma^1{}_1 = \sigma^2{}_2 = \frac{1}{3} k_3 (k_5 - k_4) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right], \quad (39)$$

$$\sigma^3{}_3 = \frac{2}{3} k_3 (k_4 - k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right], \quad (40)$$

$$\sigma^4{}_4 = 0, \quad (41)$$

$$\sigma^2 = \frac{1}{3} \left[ k_3 (k_5 - k_4) \left( \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right) \right]^2, \quad (42)$$

$$\theta = k_3 (2k_5 + k_4) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]. \quad (43)$$

From (36) and (38), we observe that the energy conditions  $\rho \geq 0$  and  $\rho_p \geq 0$  are fulfilled, provided

$$k_5(2k_4 + k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]^2 \geq \left( \frac{a}{k_3 m M} \right)^2 \left[ \frac{\xi}{L \sin(\xi \tau)} \right]^{2k_5},$$

and

$$\begin{aligned} & k_3^2 k_5 (k_4 - k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]^2 \\ & \geq \left[ k_1 k_3^2 k_5 \frac{\xi}{L \sin(\xi \tau)} - 4\pi \xi k_3 (k_4 + 2k_5) \right] \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right], \end{aligned}$$

respectively. From (37), we observe that the string tension density  $\lambda > 0$  provided

$$\begin{aligned} & 3k_3^2 k_5^2 \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]^2 + 2k_1 k_3^2 k_5 \left[ \frac{\xi^2}{L \sin(\xi \tau)} - \frac{k_1 \xi^2}{L^2 \sin^2(\xi \tau)} \right] \\ & > \left( \frac{a}{mM} \right)^2 \left[ \frac{\xi}{L \sin(\xi \tau)} \right]^{2k_5} + 8\pi k_3 \xi (k_4 + 2k_5) \left[ \xi - \frac{k_1 \xi}{L \sin(\xi \tau)} \right]. \end{aligned}$$

The model (35) represents an expanding universe when  $\sin(\xi \tau) > \frac{k_1}{L}$ . When  $\sin(\xi \tau) < \frac{k_1}{L}$ , then  $\theta$  decreases with time. Therefore the model describes a shearing non-rotating expanding universe without the big-bang start. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of the universe. Furthermore, since  $\lim_{\tau \rightarrow \infty} \frac{a}{\theta} \neq 0$ , the model does not approach isotropy for large value of  $\tau$ . However, if  $\sin(\xi \tau) = \frac{k_1}{L}$ , the model (35) represents an isotropic model in presence of bulk viscosity.

In absence of bulk viscosity, i.e., when  $\xi \rightarrow 0$ , the metric (35) reduces to

$$ds^2 = - \left( \frac{L}{k_1 k_3} \right)^2 d\tau^2 + (\tau)^{2k_5} dX^2 + e^{-\frac{2aX}{mMLk_5}} (\tau)^{2k_5} dY^2 + (\tau)^{2k_4} dZ^2. \tag{44}$$

The physical parameters  $\rho, \lambda, \rho_p$  and the kinematical parameters  $\theta, \sigma^2$  for this model are respectively given by

$$8\pi\rho = \left( \frac{k_1 k_3}{L\tau} \right)^2 k_5(2k_4 + k_5) - \left( \frac{a}{mM} \right)^2 \frac{1}{(L\tau)^{2k_5}}, \tag{45}$$

$$8\pi\lambda = \left( \frac{k_1 k_3}{L\tau} \right)^2 k_5(3k_5 - 2) - \left( \frac{a}{mM} \right)^2 \frac{1}{(L\tau)^{2k_5}}, \tag{46}$$

$$8\pi\rho_p = \left[ \frac{2k_1^2 k_3^2 k_5 (1 + k_4 - k_5)}{L^2 \tau^2} \right], \tag{47}$$

$$\sigma^1{}_1 = \sigma^2{}_2 = \frac{k_1 k_3 (k_4 - k_5)}{3L\tau}, \tag{48}$$

$$\sigma^3{}_3 = \frac{2k_1 k_3 (k_5 - k_4)}{3L\tau}, \tag{49}$$

$$\sigma^4_4 = 0, \tag{50}$$

$$\sigma^2 = \frac{1}{3} \left[ \frac{k_1 k_3 (k_4 - k_5)}{L \tau} \right]^2, \tag{51}$$

$$\theta = - \frac{k_1 k_3 (k_4 + 2k_5)}{L \tau}. \tag{52}$$

From (45) and (47), we observe that the energy conditions  $\rho \geq 0$  and  $\rho_p \geq 0$  are fulfilled, provided

$$k_5(2k_4 + k_5)(L\tau)^{2(k_5-1)} \geq \left( \frac{a}{mMk_1k_3} \right)$$

and

$$k_4 \geq k_5 - 1$$

respectively. From (46), we observe that the string tension density  $\lambda \geq 0$  provided

$$k_5(3k_5 - 2)(L\tau)^{2(k_5-1)} \geq \left( \frac{a}{mMk_1k_3} \right)^2.$$

In absence of bulk viscosity, the model (44) starts expanding with a big bang at  $\tau = 0$  and the expansion in the model decreases as time increases when  $L, 0$ . Also when  $\tau \rightarrow \infty$  then shear is zero. Near the singularity  $\tau = 0$ , the physical parameters  $\rho, \lambda, \rho_p$  are infinite if  $k_5 < 0$ . Also since  $\lim_{\tau \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$ , the model does not approach isotropy for large value of  $\tau$ .

### 4 Other Model

In general,  $\xi$  is not constant through out the fluid, so that  $\xi$  can not be taken always constant, specially when the universe is expanding. Since, in general,  $\xi$  depends on temperature ( $T$ ) and pressure( $p$ ), it is reasonable to consider  $\xi$  as function of time  $t$ .

In this case (25), after integration, leads to

$$C = \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{k_4}, \tag{53}$$

where

$$h(t) = c_0 e^{k_3 \int \xi(t) dt}. \tag{54}$$

and  $b_0, c_0$  are constants of integration. Therefore, we also obtain

$$B = M \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{k_5}, \tag{55}$$

$$A = (mM) \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{k_5}. \tag{56}$$



Hence, in this case, the metric (1) reduces to

$$dS^2 = -dt^2 + (mM)^2 \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{2k_5} dx^2 + M^2 \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{2k_5} e^{-2ax} dy^2 + \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{2k_4} dz^2. \tag{57}$$

The physical parameters  $\rho$ ,  $\lambda$ ,  $\rho_p$  and the kinematical parameters  $\theta$ ,  $\sigma^2$  for this model are respectively given by

$$8\pi\rho = N(N + 2) \left[ \frac{h(t)}{\{b_0 + k_4^{-1} \int h(t) dt\}} \right]^2 - \left( \frac{a}{mM} \right)^2 \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{-2k_5}, \tag{58}$$

$$8\pi\lambda = -\left( \frac{a}{mM} \right)^2 \left[ b_0 + k_4^{-1} \int h(t) dt \right]^{-2k_5} + \frac{N^2 [8\pi\xi \frac{(2N+1)}{N^2} + \frac{2h_1(t)}{N} \{b_0 + k_4^{-1} \int h(t) dt\} + h^2(t)(3 - \frac{1}{k_4k_5})]}{[b_0 + k_4^{-1} \int h(t) dt]^2}, \tag{59}$$

$$8\pi\rho_p = \frac{N[2(2 + \frac{1}{N})(Nh^2(t) - 4\pi\xi) + 2h_1(t)[b_0 + k_4^{-1} \int h(t) dt] - \frac{Nh^2(t)}{k_4k_5}]}{[b_0 + k_4^{-1} \int h(t) dt]^2}, \tag{60}$$

$$\theta = \frac{(2N + 1)h(t)}{[b_0 + k_4^{-1} \int h(t) dt]}, \tag{61}$$

$$\sigma^1_1 = \sigma^2_2 = \frac{(N - 1)h(t)}{3[b_0 + k_4^{-1} \int h(t) dt]}, \tag{62}$$

$$\sigma^3_3 = \frac{2(1 - N)h(t)}{3[b_0 + k_4^{-1} \int h(t) dt]}, \tag{63}$$

$$\sigma^4_4 = 0, \tag{64}$$

$$\sigma^2 = \frac{(1 - N)^2 h^2(t)}{3[b_0 + k_4^{-1} \int h(t) dt]^2}. \tag{65}$$

We observe that the equation (54) has a rich structure and admits different choice of function  $\xi(t)$ . We have to choose  $\xi(t)$  in such a manner so that (54) be integrable. Of course, the choice of  $\xi(t)$  is quite arbitrary but since we are looking for physically viable models of the universe consistent with observations, one can consider the suitable exponential, polynomial and sinusoidal form of the function  $\xi(t)$  such that (54) be integrable.

## 5 Conclusion

We have presented a new class of Bianchi type III string cosmological models in presence and absence of bulk viscosity. In our solution, we have obtained (see (21)) a relation between metric coefficients from our field equations in a natural way, while in previous researches many authors [2, 27, 28] have considered as an *ad hoc* condition to simplify their equations. If we choose  $a = -1$ , our model (33), under some particular choice of constants, gives the solution of Bali and Dave [2]. In Sect. 4, we have obtained a general solution by considering the bulk viscosity as function of time  $t$ . This general solution has a rich structure and admits many number of solutions by suitable choice of function  $\xi(t)$ . Here the choice of  $\xi(t)$  is quit arbitrary but since we look for physically viable models of the universe, one can choose  $\xi(t)$  such that (54) be integrable.

It is observed that the bulk viscosity plays significant role in the evolution of the universe. In presence of bulk viscosity the model represent an expanding, shearing and non-rotating universe without the big bang start. But, in the absence of viscosity, the model starts expanding with a big bang at  $\tau = 0$ .

**Acknowledgements** Authors would like to thank the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for providing facility and support where this work was carried out. The authors also thank to Prof. Raj Bali for fruitful discussion. The authors are thankful to anonymous referee(s) for critical remark and constructive suggestions.

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